An Excel Solver Model for a Blending Type Optimization Problem in Mining with Quadratic Programming

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Abstract: A Stochastic non-linear optimization model using Quadratic Programming (QP) is presented for a hypothetical blending type problem in mining industry. Microsoft Excel 10.0 Solver is used to develop the model for a three-seam coal mine and data are hypothetically generated for a case study problem. Optimum quantities of each run of mine (r.o.m) coal with variability in calorific values to be fed to a nearby power plant are determined with given specifications. QPs arise directly in such applications as least-squares regression with bounds or linear constraints, robust data fitting, Markowitz portfolio optimization, data mining, support vector machines and tribology. They also arise as sub problems in optimization algorithms for nonlinear programming and in stochastic optimization.

Keywords: Stochastic Optimization, Quadratic Programming, Coal Blending, Open Cast Mining

Introduction

Blending problems arise in food, feed, metals, and oil industries. The problem is to mix or blend a collection of raw materials (i.e. different types of meats, cereal grains, or crude oils) into a finished product (i.e. dog food, sausage or gasoline). The cost per unit of product is minimized and it is subject to satisfying certain quality constraints. They are usually modeled by Linear Programming (LP) and solved easily (Erarslan et al, 2001). However, a good optimizer should also exploit the correlations, the expected values, the variances and the user constraints in real optimal decision-making circumstances. In this study, another mathematical programming method is tried for a hypothetical coal mine problem, which is quadratic programming. Quadratic Programming problems (QPs) are optimization problems in which the objective function is a convex quadratic and the constraints are linear. They have the general form

$$\text{Min} \quad \tfrac{1}{2} X^T Q X + c^T X \quad \text{subject to } A X = b,\ C X >= d$$

Where Q is symmetric positive semi-definite n x n matrix, $x \in R^n$ is a vector of unknowns, A and C are matrices, and b and d are vectors of appropriate dimensions. The constraints Ax = b are referred to as equality constraints while Cx >= 0 are known as inequality constraints. QPs arise directly in such applications as least-squares regression with bounds or linear constraints, robust data fitting, Markowitz portfolio optimization, data mining, support vector machines, and tribology. They also arise as sub problems in optimization algorithms for nonlinear programming and in stochastic optimization. (Gertz and Wright, 2003).

Problem Definition

The coal with varying calorific values is mined out by open cast mining method from different production areas and is fed to a nearby power plant which has an annual capacity of 4.5 million tons and minimum heat content of 1750±100 Kcal/kg. The quality of coal seams are highly changing both horizontally and vertically, which requires a precise scheduling and blending during mining and stockpiling stages.

Otherwise, a great deal of penalty charges has to be paid by Coal Company to the power plant. The three coal seams are assumed to have mean calorific values of 2000, 1800, 1600 Kcal/kg, respectively, with equal standard deviations of 20%. It is also assumed that the coal seams are mined out selectively and stockpiled into separate stockpiles. The objective of this study is to feed the power plant requirements with the available three coal seams which are highly variable in terms of calorific values. Optimum quantities of coal to be mined from each of these three coal seams are to be determined by Quadratic Programming.

Model Construction

The key to the development of the model is the fact that for random variables, $X_1, X_2, X_3, ., X_n$

$$var(c_1 X_1 + c_2 X_2 + c_3 X_3 + \ldots + c_n X_n) = [c_1\ c_2\ c_3\ \ldots c_n]\ (\text{covariance matrix})\ [c_1\ c_2\ c_3 \ldots c_n]^T$$

The objective function in this study problem is to minimize the variance of power plants calorific values from both the upper and the lower limits (Markowitz, 1952). The constraints are capacity requirement of power plant and both upper and lower mean calorific value limits. A simple covariance matrix is assumed to represent the correlations between each coal seam (Markowitz et al., 1991). Here is how we proceed.

The steps of algorithm are presented below as:

Step 1: Trial values are entered into the changing cells as: $A$3:$C$3

Step 2: Compute the total amount of coal to be mined from three seams by the formula as

=SUM (A3:C3)

Step3: Compute the mean calorific values both for lower and upper limits by the formula as:
=SUMPRODUCT (A5:C5, A3:C3)

Step 4: The objective function to be minimized is the variance of calorific values which is computed by the formula

= MMULT (A3:C3, MMULT (A8:C10, TRANPOSE (A3:C3)))

Note that Control Shift Enter must be hit for this formula to work.

Output Results

Excel model is given in Table 1 for a hypothetical coal seam blending problem in mining industry. Excel solver model is developed in Table 2 and answer report and sensitivity reports are given in Tables 3 and 4, respectively.

B1 Seam	B2 Seam	B3 Seam	ACT_TOTAL_PROD		Plant Cap
2.062.500,0	1.500.000,0	937.500,0	4.500.000,0	=	4.500.000,0

MEAN CALORIFIC VALUES

E(X1)	E(X2)	E(X3)	MEAN_CAL_VAL		Cal_Limit
2000	1800	1600	1.650,0	>=	1650
2000	1800	1600	1.850,0	<=	1850

INPUTS	Mean Calorific Values	Standart Deviation Calorific Values
Coal Seam B1	2.000	20,0%
Coal Seam B2	1.800	20,0%
Coal Seam B3	1.600	20,0%

Correlations

	B1	B2	B3
B1	100,0%	0,0%	0,0%
B2	0,0%	100,0%	0,0%
B3	0,0%	0,0%	100,0%

Standart Deviation

B1	B2	B3
20,0%	20,0%	20,0%

Covariance Matrix

	B1	B2	B3
B1	4,0%	0,0%	0,0%
B2	0,0%	4,0%	0,0%
B3	0,0%	0,0%	4,0%

CAL_VAL_VAR
2,95313E+11

Table 1. An Excel Solver Model for Coal Seam Blending Problem

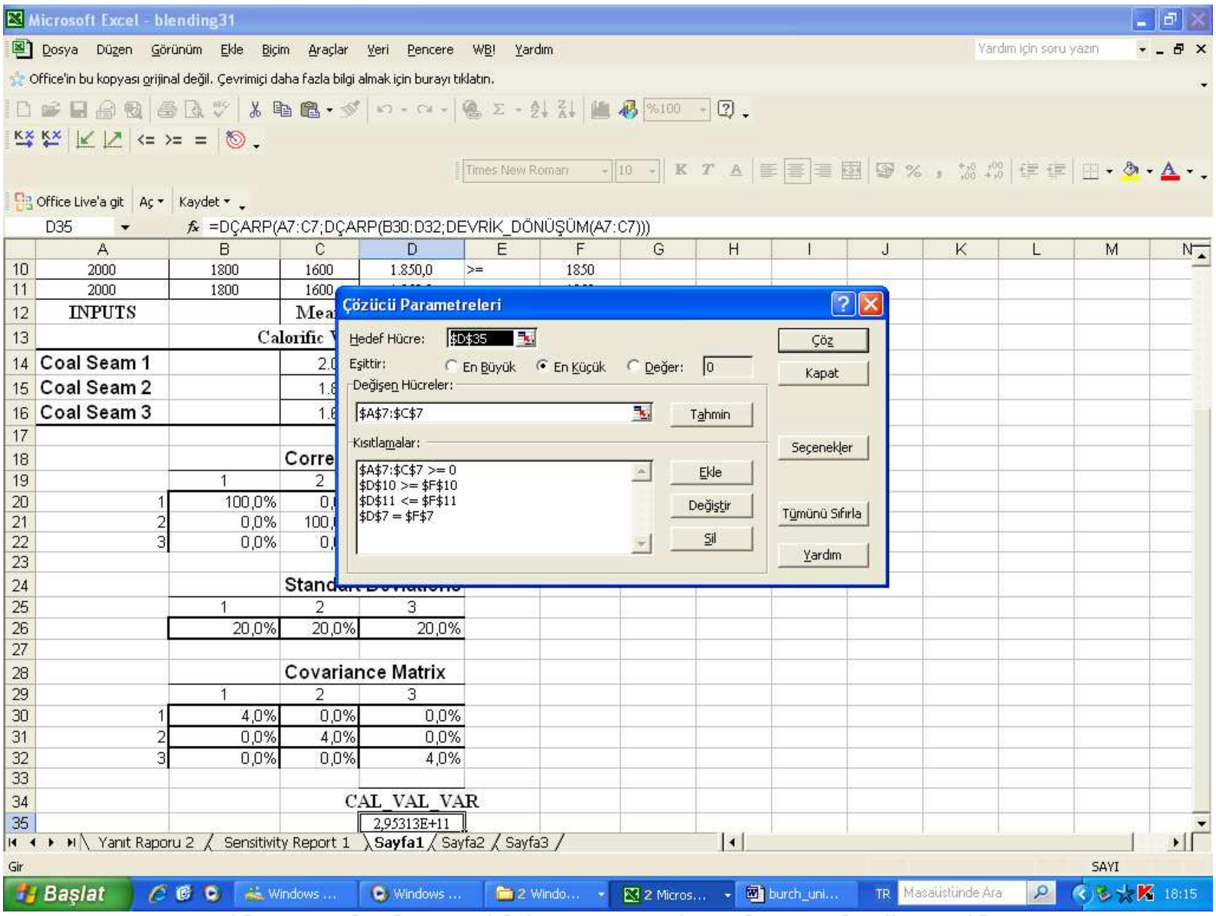

Table 2. Excel Solver Model for a Case Study Coal Seam Blending Problem

Microsoft Excel 10.0 Yanıt Raporu
Çalışma Sayfası: [blending3.xls]Sayfa1
Yaratılan Rapor: 18.03.2009 13:27:43

Hedef Hücre (En Küçük)

Hücre	Ad	İlk Değer	Son Değer
$D$35	CAL_VAL_VAR	2,16738E+12	2,35723E+12

Ayarlanabilir Hücreler

Hücre	Ad	İlk Değer	Son Değer
$A$7	B1 Seam	2.062.500,0	2.249.999,9
$B$7	B2 Seam	1.500.000,0	1.687.500,1
$C$7	B3 Seam	937.500,0	562.500,0

Sınırlamalar

Hücre	Ad	Hücre Değeri	Formül	Durum	Serbestlik
$D$10	MEAN_CAL_VAL	1.650,0	$D$10>=$F$10	Aynı	0,0
$D$7	ACT_TOTAL_PROD	4.500.000,0	$D$7=$F$7	Farklı	0
$A$7	B1 Seam	2.249.999,9	$A$7>=0	Farklı	2.249.999,9
$B$7	B2 Seam	1.687.500,1	$B$7>=0	Farklı	1.687.500,1
$C$7	B3 Seam	562.500,0	$C$7>=0	Farklı	562.500,0

Table 3. Answer Report for Case Study Coal Seam Blending Problem

Microsoft Excel 10.0 Sensitivity Report
Worksheet: [blending31.xls]Sayfa1
Report Created: 14.05.2009 23:00:22

Adjustable Cells

Cells	Name	Final Value	Reduced Gradient
$A$7	B1 Seam	2.062.500,0	0,0
$B$7	B2 Seam	1.500.000,0	0,0
$C$7	B3 Seam	937.500,0	0,0

Constraints

Cells	Name	Final Value	Lagrange Multiplier
$D$10	MEAN_CAL_VAL	1.850,0	1.012.500.930,5
$D$11	MEAN_CAL_VAL	1.850,0	0,0
$D$7	ACT_TOTAL_PROD	4.500.000,0	131.250,1

Table 4. Sensitivity Report for a Case Study Coal Seam Blending Problem

In a real case study, sufficient amount of data should be collected for representing the probability density functions for each coal seam and their relationships between them for more accurate results. Geostatistical analysis is required to model the variability for each coal seam with proper variograms.

Conclusions

In this study, a new approach to the blending problem of open pit coal mines is modeled and solved by Quadratic Programming method. Optimum coal blends satisfying the needs of a hypothetical power plant are determined. The models are developed and solved in Excel Solvers 10.0 and it predicts reasonably well for multiple coals with varying calorific values both horizontally and vertically. The developed model can easily be modified for many seams situations other than three seams.

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