A Critique On The Consistency Ratios Of Some Selected Articles Regarding Fuzzy Ahp And Sustainability

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Abstract

Consistency ratio (CR) is a very important indicator for achieving the reliability of an individual's pairwise comparisons in Analytic Hierarchy Process (AHP). Although the applications of fuzzy AHP need this kind of CR results as well, only a few of studies include these results. The most accepted method to calculate CR for fuzzy pairwise comparison matrices (PCMs) is to transform fuzzy numbers to crisp versions and to proceed as in the ordinary CR calculations of AHP. Triangular fuzzy numbers (TFNs) are usually used to present linguistic terms of an individual's pairwise comparisons. In this research, CRs of 242 PCMs presented with TFNs, found in 39 articles, have been calculated based on four widely used defuzzification methods. The aim of this research is to find out if the PCMs of some available articles regarding sustainability issues in literature are reliable. After CR calculations of those PCMs, it has been found that some of them are reliable while many others are not. After reviewing these findings, researchers in fuzzy AHP field are expected to give much attention to those CR issues and try to obtain PCMs that are more reliable.

Keywords: fuzzy AHP, consistency ratio, sustainability, defuzzification, fuzzy numbers

1.INTRODUCTION

Analytic Hierarchy Process (AHP), developed by Saaty (1980), is a very powerful multicriteria prioritization technique. However, its applicability is not only limited to the number of criteria but also to the consistency ratio (CR) of pairwise comparison matrices (PCMs). If the CR value of a PCM passes 0.1, it means that the matrix is not consistent and comparisons should be made again in order to have a reasonable CR value.

Fuzzy AHP is developed because of the fuzzy nature of those pairwise comparisons (Chang 1996). To provide more fuzziness (according to Saaty and Tran's '2007' criticism), usually triangular fuzzy numbers (TFNs) are used in pairwise comparisons. Although some kind of CR value is needed to evaluate those PCMs in fuzzy AHP as well, many of the research articles skip this evaluation. The aim of this study is to present how much the results of those researches are reliable when taking CR issues into consideration. A total of 242 PCMs, found in 39 articles, have been examined in terms of their crisp version CR values.

The paper is organized as follows: first, the applications of fuzzy AHP in literature are described; second, some defuzzification methods for obtaining the crisp versions of those TFNs are explained; third, how TFNs differ in their crisp versions under different diffuzzification methods are shown; fourth, CR values of PCMs in some selected articles are evaluated. Finally, remarkable conclusions and some future directions are given.

2.FUZZY AHP AND ITS APPLICATIONS IN LITERATURE

Fuzzy AHP is one of the most widely used methods in multicriteria decision making although Saaty and Tran (2007) criticizes it seriously about its fuzzifying judgments. Some applications of fuzzy AHP exist in the field of sustainability and sustainable developments. Although there are some fuzzy approaches to obtain a priority weight vector (Chang 1996, Liou and Wang 1992), Chang's extent synthesis method is used in many fuzzy AHP studies.

Among many research areas regarding sustainability and fuzzy AHP, some of them can be given as follows:

- Supplier or firm selection: Kahraman et al. (2004), Chan et al. (2008), Efendigil et al. (2008), Lee et al. (2009), Şen et al. (2010), Chen et al. (2011), Pei et al. (2011), Aydin et al. (2012), Öztürk and Başkaya (2012).
- Production process selection: Talinli et al. (2010).
- Market selection: Deng and Molla (2008), Toksarı and Toksarı (2011).
- Facility location selection: Ertuğrul and Karakaşoğlu (2008), Kayikci (2010).
- Resource allocation: Ahari et al. (2011), Bulut et al. (2012).
- Personnel selection: Celik et al. (2009), Pei et al. (2011), Bulut et al. (2012).
- Quality issues: Kwong and Bai (2003), Büyüközkan et al. (2011), Aydin et al. (2012).
- Strategy prioritization: Lin et al. (2010), Chaghooshi et al. (2011).

- Environmental issues: Lee et al. (2009), Tseng et al. (2009), Zheng et al. (2010), Karimi et al. (2011), Wang et al. (2011).
- Some other managerial issues: Zheng (2011), Bulut et al. (2012).

3.DEFUZZIFICATION OF TRIANGULAR FUZZY NUMBERS

A TFN can be defined by the membership function (1) (Bulut *et al*. 2012).

$$\mu_{\tilde{A}}(x)=\begin{cases}0, & x<l,\\ (x-l)/(m-l), & l\le x<m,\\ 1, & x=m,\\ (u-x)/(u-m), & m<x\le u,\\ 0, & u<x,\end{cases} \tag{1}$$

where l and u correspond to the lower and upper bounds of the fuzzy number $\tilde{A}$, respectively, and m is the midpoint. The TFN is indicated as $\tilde{A} = (l, m, u)$.

The methods for defuzzification of TFNs used in this study are as follows:

- **Weighted Mean Method:** According to Kwong and Bai (2003), a TFN can be defuzzified to a crisp number by equation (2).

$$\tilde{A}\_crisp=\frac{(l+4m+u)}{6} \tag{2}$$

  This method has been used by some researchers for transforming TFNs to crisp versions and calculated CRs accordingly. Although equation (2) is very easy to calculate, it may cause some bias problems because of weighting. It is more appropriate for the TFNs shaping as an equilateral triangle. However, especially the reciprocals of TFNs used in fuzzy PCMs do not shape as an equilateral triangle most of the time.

- **Centroid Method:** It is also called "center of gravity" method and is the most widely used one among other defuzzification methods. Centroid defuzzification returns the center of area under the curve as in equation (3).

$$\tilde{A}\_crisp=\frac{\int \mu_{\tilde{A}}(x).xdx}{\int \mu_{\tilde{A}}(x)dx} \tag{3}$$

  For TFNs as in function (1), the result of centroid method will be equal to $(l + m + u)/3$.

- **Bisector Method:** The bisector is the vertical line that will divide the region into two sub-regions of equal area. It will be equal to m for equilateral TFNs.

- **Middle, Smallest, and Largest of Maximum (MOM, SOM, and LOM) Methods:** Since any TFN used in a PCM has a unique maximum, the result of these three methods will be the same. Therefore, only the MOM results have been shown in this study.

4.VARIATIONS IN TRIANGULAR FUZZY NUMBERS

The TFNs used for importance degrees are equilateral linguistic variables except for the highest level. In some researches such as Zheng *et al*. (2010), Zheng (2011), Wang *et al*. (2011), Tseng *et al*. (2009), Toksarı and Toksarı (2011), Bozbura and Beskese (2007), Ahari *et al*. (2011), Büyüközkan *et al*. (2011), the highest level is also equilateral. However, in many other researches, the highest level is not equilateral. On the other hand, reciprocal values of any TFNs are not equilateral in almost every time. Because of these reasons, weighted mean method is not the best approach in many situations and researchers need to look at how some other defuzzification methods will result. Table 1 shows how crisp versions of some reciprocal TFNs can differ in terms of different defuzzification techniques.

Table 1. Defuzzification of TFNs

		Crisp Versions Resulting from Defuzzification Methods			
Importance Degrees*	TFNs	Weighted Mean	Centroid	Bisector	MOM
Equally important	(1/2, 1, 3/2)	1.0000	1.0000	1.0000	1.0000
Weakly more important	(1, 3/2, 2)	1.5000	1.5000	1.5000	1.5000
Strongly more important	(3/2, 2, 5/2)	2.0000	2.0000	2.0000	2.0000
Very strongly more important	(2, 5/2, 3)	2.5000	2.5000	2.5000	2.5000
Absolutely more important	(5/2, 3, 7/2)	3.0000	3.0000	3.0000	3.0000
	Reciprocals				
Reciprocal of equally important	(2/3, 1, 2)	1.1111	1.2222	1.1835	1.0000
Reciprocal of weakly more important	(1/2, 2/3, 1)	0.6944	0.7222	0.7113	0.6667
Reciprocal of strongly more important	(2/5, 1/2, 2/3)	0.5111	0.5222	0.5176	0.5000
Reciprocal of very strongly more important	(1/3, 2/5, 1/2)	0.4056	0.4111	0.4087	0.4000
Reciprocal of absolutely more important	(2/7, 1/3, 2/5)	0.3365	0.3397	0.3383	0.3333

Notes: * These can be found in Zheng *et al*. (2010), Zheng (2011), Wang *et al*. (2011), Tseng *et al*. (2009), Toksarı and Toksarı (2011), Bozbura and Beskese (2007).

5.EVALUATIONS OF SOME SELECTED ARTICLES

A total of 39 articles regarding fuzzy AHP and sustainability have been taken from the literature. There are 242 PCMs developed with TFNs in those articles. A MATLAB m_file has been coded to compute all of those PCMs' CRs according to above mentioned four defuzzification methods. Table 2 shows just two of those articles' calculation results. It is not

possible to show all CR values for all PCMs in this space limited study. On the other hand, the articles and their number of PCMs that have CR values above 0.1 are shown in Table 3.

Table 2. Some examples of exact CR calculation results

Articles	PCMs Presented with TFNs (Matrix No.)	CR Values based on The Following Methods			
		Weighted Mean	Centroid	Bisector	MOM
Chen *et al*. (2011)	1	0.0432	0.0771	0.0637	0.0089
	2	0.0965	0.1655	0.1464	0.0176
	3	0.1566	0.1826	0.1724	0.1304
	4	0.3750	0.3950	0.3878	0.3533
Lee *et al*. (2009)	1	0.0761	0.0988	0.0912	0.0526
	2	0.1573	0.2216	0.2006	0.0910

Table 3. CR results of matrices in literature

Articles	Number of PCMs Presented with TFNs	Number of PCMs whose Crisp Versions Have a CR above 0.1 based on The Following Defuzzification Methods			
		Weighted Mean	Centroid	Bisector	MOM
Ahari *et al*. (2011)	2	-	1	1	-
Aydin *et al*. (2012)	1	1	1	1	1
Bulut *et al*. (2012)*	25	14	19	19	3
Büyüközkan *et al*. (2011)*	26	-	2	1	-
Celik *et al*. (2009)	6	4	4	4	2
Chaghooshi *et al*. (2011)*	4	1	2	1	-
Chan *et al*. (2008)	8	1	1	1	1
Chen *et al*. (2011)	4	2	3	3	2
Deng and Molla (2008)	5	5	5	5	4
Efendigil *et al*. (2008)	1	1	1	1	1

Ertuğrul and Karakaşoğlu (2008)	6	6	6	6	6
Kahraman *et al*. (2004)	15	9	10	9	9
Karimi *et al*. (2011)	5	1	1	1	1
Kayikci (2010)	1	1	1	1	1
Kwong and Bai (2003)*	9	2	3	3	-
Lee *et al*. (2009)*	2	1	1	1	-
Lee *et al*. (2011)*	5	2	3	3	2
Lin *et al*. (2010)	6	5	6	6	-
Öztürk and Başkaya (2012)*	19	7	14	11	-
Pei *et al*. (2011)	21	18	20	20	9
Şen *et al*. (2010)	4	-	2	1	-
Talinli *et al*. (2010)	1	1	1	1	1
Toksarı and Toksarı (2011)	8	5	5	5	5
Tseng *et al*. (2009)*	5	3	3	3	3
Wang *et al*. (2011)	4	1	1	1	-
Zheng (2011)	3	-	2	1	-
Zheng *et al*. (2010)	7	-	1	-	-

Notes: * Some CRs or related explanations are presented inside the article.

6.CONCLUTIONS

Many researchers continue to use fuzzy AHP without any explanation and calculation about CRs of PCMs used in their studies. Although the methods and applications submitted in their researches give valuable contributions to literature, violation of the CR limit may decrease the reliability of their currently published articles.

In this study, after a broad review of literature, 242 PCMs in 39 articles have been examined. The TFNs have been transformed into their crisp versions via four different defuzzification methods and the CRs of those PCMs have been calculated as in the ordinary AHP. The results have shown that many of those PCMs violated the 0.1 upper-limit under some defuzzification methods while they did not violate this limit under some others. Because of this reason, researchers in fuzzy AHP field need to pay much attention to the CR issues for their PCMs. More researches are needed to formulate a more suitable CR for fuzzy AHP applications.

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