Synchronization of a chaotic finance system via active control

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Abstract

This paper discusses chaos synchronization of the three dimensional finance system based on active control technique. Using active control theory, chaos synchronization of three dimensional chaotic finance system is realized with three input. The designed controllers ensure the stability of error dynamical system between two identical chaotic finance systems. Also, the controllers provide that the error dynamical system converges to zero equilibrium. Numerical simulations show that the proposed method is effective for chaotic finance system.

Keywords: Chaotic finance system, chaos synchronization, active control

1. INTRODUCTION

Since the control of chaotic systems is firstly proposed by Ott, Grebogi and Yorke, chaos control has become one of the much interesting research subject. Also, chaos synchronization has received a huge increasing interest and has been studied in the past two decades, after Pecora and Carroll introduced the synchronization method. Recently, many control methods are proposed to the control and the synchronization of the chaotic systems. The control strategies applied to control and synchronization of chaos such as OGY method (E. Ott, C. G. 1990), linear feedback control (A.E. Matouk, 2008), passive control (S. Emiroğlu and Y. Uyaroğlu, 2010; X. Chen, C. L. 2010), active control (S. Emiroğlu, Y. Uyaroğlu, 2011) etc..

In this paper, we study the control of chaos in a nonlinear finance chaotic system which was proposed by reference (Guoliang Cai, 2007). The state equations of chaotic finance system are written below Eq 1. (Guoliang Cai, 2007)

$$\begin{aligned} \dot{x} &= z + (y - a)x \\ \dot{y} &= 1 - by - x^2 \\ \dot{z} &= -x - cz \end{aligned} \tag{1}$$

where variable x represents the interest rate in the model; variable y represents the investment demand and variable z is the price exponent. The parameter a is the saving. b is the per-investment cost. c is the elasticity of demands of commercials. And they are positive constants.

Mathematical model of a finance system is constructed by using Matlab-Simulink program as shown in Figure 1.

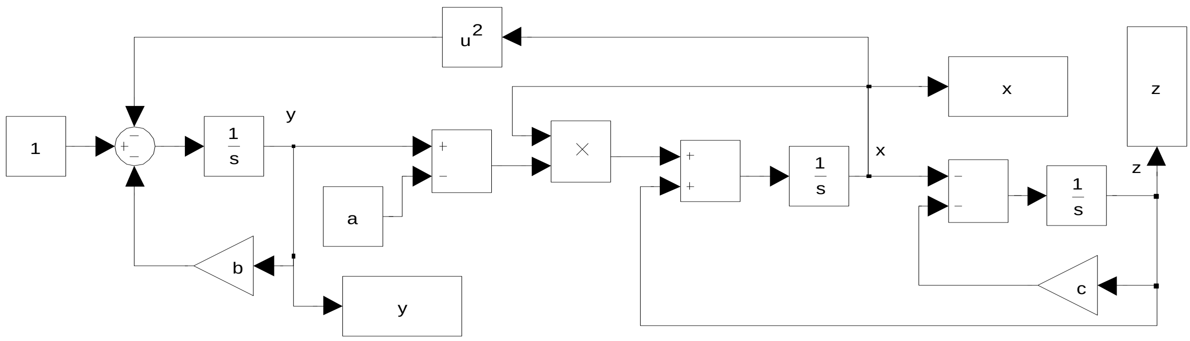

Figure 17. Matlab-Simulink model of finance system

Chaotic time series and phase portraits of the system are shown in Figure 2.

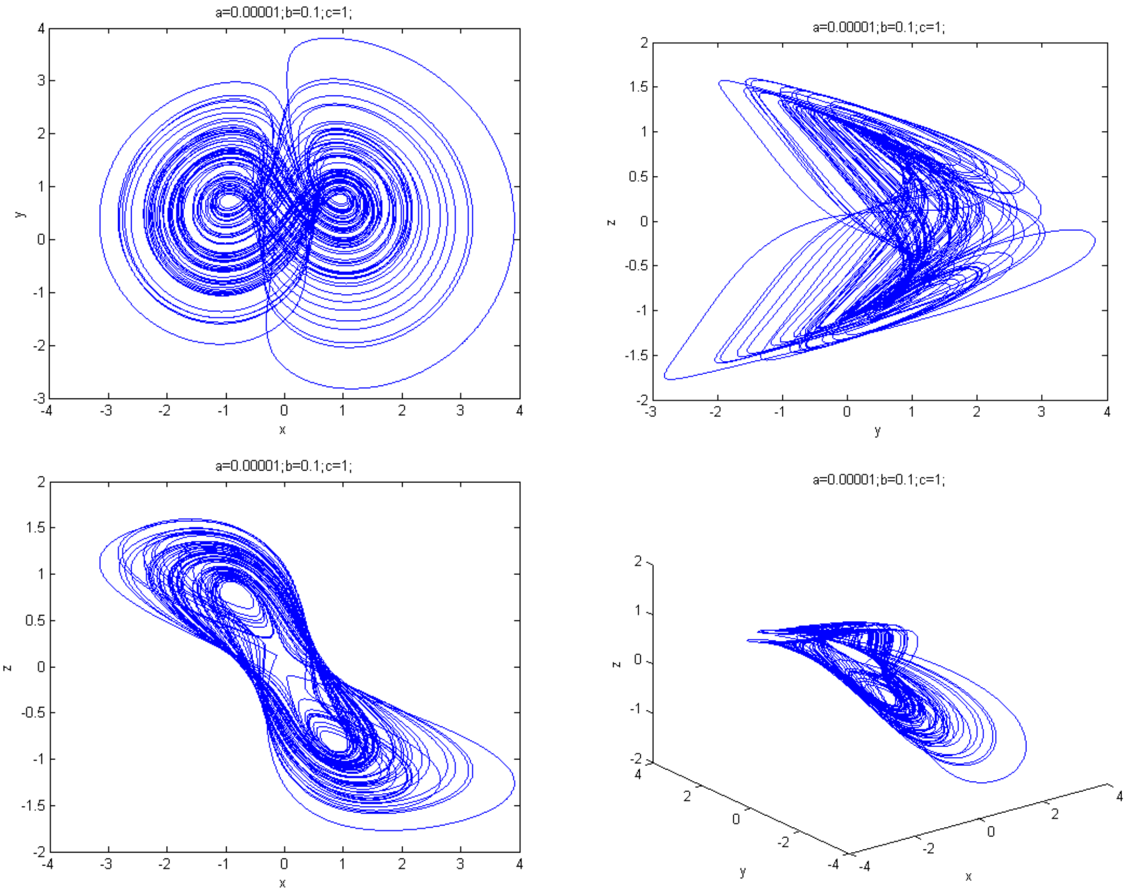

Figure 18. Phase portraits of the system

2. SYNCHRONIZATION OF CHAOTIC FINANCE SYSTEM

In this section, the synchronization of finance chaotic system (2) is achieved using active control theory. The active control scheme is employed to realize chaos synchronization. Suppose the drive system below,

$$\begin{aligned} \dot{x}_1 &= z_1 + (y_1 - a)x_1 \\ \dot{y}_1 &= 1 - by_1 - x_1^{\ 2} \\ \dot{z}_1 &= -x_1 - cz_1 \end{aligned} \tag{2}$$

and the response system,

$$\begin{aligned} \dot{x}_2 &= z_2 + (y_2 - a)x_2 + u_1 \\ \dot{y}_2 &= 1 - by_2 - x_2^{\ 2} + u_2 \\ \dot{z}_2 &= -x_2 - cz_2 + u_3 \end{aligned} \tag{3}$$

where ui(i = 1,2,3) is active control function to design.

Then the error dynamical system between the drive system (2) and the response system (3) is

$$\begin{aligned}
\dot{e}_1 &= e_3 + y_2 x_2 - y_1 x_1 - a e_1 + u_1 \\
\dot{e}_2 &= -b e_2 - x_2^{\;2} + x_1^{\;2} + u_2 \\
\dot{e}_3 &= -e_1 - c e_3 + u_3
\end{aligned} \tag{4}$$

in which e1 = x2 - x1, e2 = y2 - y1 and e3 = z2 - z1.

The active control functions can be designed as

$$\begin{aligned}
u_1 &= -e_3 - y_2 x_2 + y_1 x_1 + (a-1) e_1 \\
u_2 &= (-b-1) e_2 + x_2^{\;2} - x_1^{\;2} \\
u_3 &= e_1 + (c-1) e_3
\end{aligned} \tag{5}$$

When the controllers are added to chaotic system, then the eigenvalues of the closed loop system are -1, -1 and -1. Hence, in light of the linear system theory, our choice can ensure that the origin of the error dynamical system is asymptotically stable.

Numerical simulation of synchronization of chaotic finance system which has different initial conditions is performed. The initial conditions of the drive system and the response system are (0.1 0.23 0.3) and (0.1 0.5 0.7) respectively. Figure 3 shows time series of drive and response system and error between drive and response system. As can be seen from Figure 3, after controllers are activated t=50s, the synchronization of two chaotic finance system with different initial conditions is realized and also, can be seen that the error between drive and response system converges to zero.

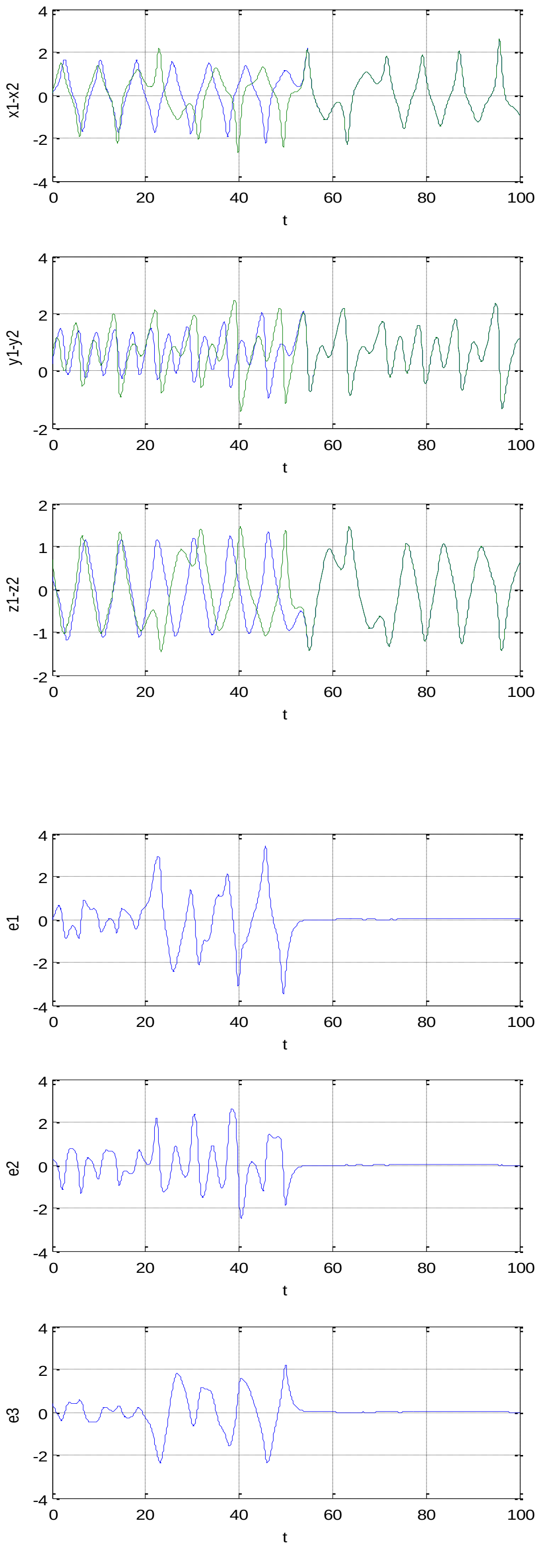

Figure 3 Time series of the drive and response system and error between drive and response system when active controllers are activated at t=50s

3. CONCLUSION

This work discusses chaos synchronization of the finance chaotic system. Synchronization between the two finance chaotic systems with different initial conditions is achieved by using active control scheme. Finally, numerical simulations are provided to verify the theoretical analysis and also show that the proposed method works effectively.

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