# A Model for Discrete Time/Space Approximation of the Vasicek Model for the Interest Rate

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In this paper we present the mathematical model for the real interest rate as an autoregressive discrete time and discrete state space process. The process is an approximation of Vasicek continuous time–space autoregressive process presented in Vasicek (1977). We choose Vasicek model for interest rate for developing bond prices as the one which is used in the analysis of optimal asset allocation problems by many authors. It is a type of one factor short rate model where interest rate movements are driven by one source of market risk. Our model can be used in many applications when modelling an interest rate mathematically or for making simulations on the computer. The shortcoming of Vasicek model is the positive probability of the negative value of interest rate. Due to mean reverting characteristic of the interest rate, even for the negative value of real interest rate, there will be a certain demand for both traditional and index–linked bonds. It is possible to derive the bond market model using the interest rate which does not allow the negative values of the interest rate, for example Cox–Ingersoll–Ross model (Cox et al (1985)). Although CIR model may be more appropriate, and the one and ten years rolling bonds market model can be developed using CIR model, it would be also computationally more demanding. In our model we assume that the discrete time interval is one year. We will show below the technique to transform the continuous time Vasicek process into a discrete time one. We assume that the interest rate can take a finite number of values in a reasonable range. As the Vasicek process transformed into discrete time is still a continuous state space process we use the technique from Tauchen and Hussey (1991) and as a result we get a process with discrete time–state space. Once we obtain a discrete time–state process for real interest rate we can model bond prices as the expected present value of future incomes from the bond. As we assume a zero coupon bond, it means that the bond price is expected present value of one money unit that will be due in *n* years' time, where *n* years is the bond duration. Following the Vasicek approach, we can also introduce a market price of risk. As a final result we get the approximation of the bond market.

**Keywords:** Discrete Time, the Vasicek Model, Interest Rate**.**