MULTI-RESOLUTION WAVELET ANALYSIS FOR FAULT DETECTION

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Abstract

In this study, a multi-resolution wavelet analysis technique is applied to simulation data for fault detection. Data is simulated at the MATLAB environment. For this purpose, a sinusoidal wave form is generated at around 1 kHz sampling frequency and then a faulty case is simulated between 250- 500 Hz using a random process under the band-pass filtering. Hence data and its noisy form are used to show healthy and faulty cases of any physical system respectively. In order to show the fundamental properties of the data set, power spectral density variations are shown to indicate the availability of the data. After that Multi–Resolution Wavelet Analysis (MRWA) is applied to each case. In general, wavelet transform is a time-scale analysis technique which can be accepted as an alternative method to the Fourier transform. However, in this study, MRWA approach is considered. MRWA is a kind of the discrete wavelet transform and it uses filter banks approach. Hence, the time domain properties are shown in the sense of the statistical parameters. Also, calculating the power spectral densities, this comparison is done in frequency domain. With this way, a faulty case and its some properties can be determined at both of the time and frequency domains.

Key Words*:* Wavelets, Filtering, Sub-band analysis, Fault detection

Introduction

Anomaly is an unwanted transient case in the system which occurs in very short time in the signal and can be detected from the signal characteristics. Anomalies in data can be translated to significant information in a wide variety of application domains for this reason; this translation method can be named as anomally detection in general. Detection of outliers or anomalies were started to be studying in the 19th century (Edgeworth, 1887). And its results can be very important in terms of the system reliability and economical operation of the some critical systems related with energy production, space applicatons and so on.

Anomalies might be caused because of such a terrorist activity, credit card fraud, cyber-terrorism, malicious threats or breakdown of a system, e.g. Noise removal (Teng, Chen and Lu, 1990) and noise accommodation (Rousseeuw and Leroy, 1987) are deal with unwanted noise and related with anomaly detection(Chandola, Banerjee and Kumar, 2009). Noisy data considered is as an obstacle to analysis and that is the reason why it is of interest to analysts, meaning that they are responsible to clean the data before analysis in order to get useful information out of them.

Noise reduction is necessary before any data analysis is performed on the data to wipe out the unwanted objects. Towards anomalous observations, noise accommodation mentions about self-defense of a new model of estimation (Huber, 1974). Novelty detection (Markou and Singh, 2003; Saunders and Gero, 2000) whose goal is to detect previously unrealized (emergent, novel) patterns in the data, is also related with anomaly detection. Not being added into the initial model after detection is the main difference of novel patterns and anomalies. Another research on signal and noise separation in time series is studied by Khelifa, Kahlouche and Belbachir (2012). Two approaches are used to check the noises which are the wavelet transform in the frequency space and the Singular Spectrum Analysis (SSA) in the phase space. By this process the main goal is extracting the noise from signals and wavelet analysis is found as more rapid and direct for the determination of noise.

In this paper we dealed with these problems and it is prepared to provide a structured and comprehensive overview of the research on anomaly detection with the artificial data generation in MATLAB environment. There are various methods to detect the anomaly according to the signal in the data. Under the assumptions to be considered in this paper:

The Linear sytems provide the super-position principle and most simple case of the signals/ sytems can be accepted as linear time-invariant signals. Deterministic signals can be defined by analytical functions, Random Signals can be defined by means of the probability distribution functions using the random variable concept. Any anomally case, which will occur in the system, can be detected from the signal characteristics.For this purpose, there are so many mathematical approaches. In this area, several methods can be shown by the following items :

1. Statistical Calculations
2. Spectral Analysis methods like FourierTransform
3. Time-Frequency analysis like Short-Time Fourier Transform
4. Time-Scale analysis like Wavelet transform.

In this study, we considered linear, deterministic/Random, non-stationary signal types and we used short-time fourier transform based on time frequency domain and wavelet analysis as an anomaly detection techniques. Wavelet methods facilitate to zoom into the details and draw a comprehensive picture of the time series in different scales. It provides to detect and isolate the anomalies. The failure or fault detection methods are similar with the anomaly detection methods, and also, they can be described as a transient case which occurs in very short time in the signal. For this reason, it can be named as anomally detection in general. For this purpose, we will produce deterministic signal like pure sinosoidal or any signal with harmonics.

In terms of the simulation of the anomaly case, we used random signal characteristics and we produced random number in standard normal distribution. After that changing the statistical parameters or statistical properties of the randomness, we considered the different random signal characteristics. Also, in terms of the frequency domain properties, we used the band-pass filters to generate the data in a special frequency band.

In this paper, there are two important aproaches. These are as folows:

1. Detection of the anomally
2. Isolation of the transient case.

From this view point, for the detection case, we considered the Fourier Transform based applications like Short-Time Fourier Transform. In this manner, we tried to find the most suitable technique for the non-stationary signals. Then the anomaly case was isolated from the data by the Multi-Resolution Wavelet Analysis (MRWA). In this study we used Wavelet analysis but a thorough presentation of Fourier analysis is provided as well. Because the Fourier methods are an alternative for the wavelet methods and although there are different methods of wavelets, all of them are based on Fourier analysis (Mallat, 1999).

Wavelet Transforms and Multi Resolution Analysis

Wavelets are functions are used to represent data or functions and satisfy certain mathematical requirements. Thus the Wavelet transform can be used to decompose a signal into different frequency components and then present each component with a resolution matched to its scale. In the signal analysis framework, the Wavelet transform of the time varying signal depends on the scale that is related to frequency and time. Hence, the Wavelets provide a tool for time-frequency localization. The main idea behind wavelets is to analyze according to scale. Therefore, wavelet algorithms can process data at different scales or resolutions. This concept of signal analysis is termed Multi-Resolution Analysis (MRA) and it makes the Wavelets interesting and useful.

Wavelet Transforms

In 1909, Haar first mentioned about the wavelets which had a compact support means that itvanishes outside of the finite interval, but Haar wavelets are not continuously differentiable. Later wavelets are considered with an effective algorithm for numerical image processing by an earlier discovered function that can vary in scale and can conserve energy when computing the functional energy (Gabor, 1946). Between 1960 and 1980, mathematicians such as Grossman and Morlet (1985) defined wavelets in the context of quantum physics. Mallat (1989) gave a boost to digital signal processing by inventing the pyramidal algorithms, and orthonormal wavelet bases. Later Daubechies (1990) used Mallat's work to construct a set of wavelet orthonormal basic functions that are the cornerstone of wavelet applications today. The class of functions that present the wavelet transform are those that are square integrable on the real time. This class is denoted as $L^2(R)$

(1)

The mother wavelet is scaled and translated in the wavelet analysis to generate the set of functions.

The wavelet function $\psi(x) \in L2(R)$ consists of two parameters which vary continuously, they are known as dilation (a) and translation (b). A wavelet basic functions $\psi_{a,b}(x)$ is given as

$$\psi_{a,b}(x) = \frac{1}{\sqrt{|a|}}\psi(\frac{x-b}{a}) \quad a,b \in R; a \neq 0 \tag{2}$$

Here, the location of the wavelet in time is measured by the translation parameter, "*b*". The "narrow" wavelet can attain high frequency information, while the more widened wavelet can attain low frequency information. Hence the parameter "*a*" differs for different frequencies. The continuous wavelet transform is defined by

$$W_{a,b}(f) = < f, \psi_{a,b} > = \int_{-\infty}^{+\infty} f(x)\psi_{a,b}(x)dx. \tag{3}$$

The wavelet coefficients are assigned as the inner product of the function that is transformed with each basis function. Daubechies (1990) conceived one of the most sophisticated families of wavelets, named Compactly Supported Orthonormal Wavelets, and are used in Discrete Wavelet Transform (DWT). The scaling function is used to calculate the ψ in this approach. It is defined by:

$$\phi(x) = \sum_{k=0}^{N-1} c_k \phi(2x-k) \tag{4}$$

And its corresponding wavelet ψ *(x) is* defined by:

$$\psi(x) = \sum_{k=0}^{N-1} (-1)^k c_k \phi(2x+k-N+1), \tag{5}$$

Here *N* corresponds to an even number of wavelet coefficients *ck*, k = 0 to N-1. Dilation and translation of signal function $\psi(x)$ provides the discrete representation of a wavelet basis of $L^2(R)$ which is orthonormal compactly supported. If we assume that to dilation parameters "*a*" and "*b*" are assigned only discrete values:

$$a = a_0^{\ j}, b = kb_0 a_0^{\ j}, \quad \text{where } k, j \in \square, \quad a_0 > 1, \text{ and } b_0 > 0.$$

Than the wavelet function could be written as follows:

$$\psi_{j,k}(x) = a_0^{-j/2}\psi(a_0^{-j}x - kb_0) \tag{6}$$

And we have the Discrete-Parameter Wavelet Transform (DPWT) to be:

$$DPWT(f) = <f, \psi_{j,k}> = \int_{-\infty}^{+\infty} f(x) a_0^{-j/2} \psi(a_0^{-j} x - k b_0) dx \tag{7}$$

In order to make the analysis efficient and accurate, the choice between dilations and translations is made on the basis of the power of two. The frequency axis is divided into band by using the power of two for the scale parameter $''a''$.

Considering samples at the dyadic values, we have $b_0 = 1$ and $a_0 = 2$, so, the discrete wavelet transform is

$$DPWT(f) = <f, \psi_{j,k}> = \int_{-\infty}^{+\infty} f(x) \left\{ 2^{-j/2} \psi(2^{-j} x - k) \right\} dx \tag{8}$$

and $\psi_{j,k}(x)$ is defined as

$$\psi_{j,k}(x) = 2^{-j/2} \psi(2^{-j} x - k), \qquad j,k \in \mathbb{Z} \tag{9}$$

Multi-resolution Analysis (MRA)

An efficient algorithm is introduced in1989 by Mallat which perform the DPWT known as the Multi-Resolution Analysis(MRW).It is well known in signal processing area as the Two-Channel Sub-Band Coder.The MRA of $L^2(R)$ consists of successive approximations of the space V_j of $L^2(R)$.A scaling function $\phi(x) \in V_0$ exists such that $\phi_{j,k}(x) = 2^{-j/2} \phi(2^{-j} x - k)$; $j,k \in Z$ (10)

For the scaling function $\phi(x) \in V_0 \subset V_1$, there is a sequence $\{h_k\}$,

$$\phi(x) = 2 \sum_k h_k \phi(2x - k) \tag{11}$$

This equation is known as the two-scale difference equation. Furthermore, let us define W_j as a complementary space of V_j in V_{j+1}, such that $V_{j+1} = V_j \oplus W_j$ and $\bigoplus_{j=-\infty}^{+\infty} W_j = L^2(R)$. Since the $\psi(x)$ is a wavelet and it is also an element of V_0, a sequence $\{g_k\}$ exists such that

$$\psi(x) = 2 \sum_k g_k \phi(2x - k) \tag{12}$$

It is concluded that the multiscale representation of a signal $f(x)$ may be achieved in different scales of the frequency domain by means of an orthogonal family of functions $\phi(x)$.Now, let us see how the function in V_j is computed.The projection of the signal $f(x) \in V_0$ on V_j defined by $P_v f^i(x)$ is given by

$$P_v f^i(x) = \sum_k c_{j,k} \phi_{j,k}(x) \tag{13}$$

Here, $c_{j,k} =< f,\phi_{j,k}(x) >$. Similarly, the projection of the function $f(x)$ on the subspace W_j is also defined by $P_v f^i(x) = \sum_k d_{j,k}\psi_{j,k}(x)$ (14)

where $d_{j,k} =< f,\psi_{j,k}(x) >$. Because $V_j = V_{j-1} \oplus W_{j-1}$, the original function $f(x) \in V_0$ can be rewritten as

$$f(x) = \sum_k c_{j,k}\phi_{j,k}(x) + \sum_j^{J-1}\sum_k d_{j,k}\psi_{j,k}(x),\ J > j_0 \quad (15)$$

The coefficients $c_{j-1,k} = \sqrt{2}\sum_i h_{i-2k}c_{j,k}$ (16)

and

$$d_{j,k} = \sqrt{2}\sum_j g_{j-2k}c_{j,k} \quad (17)$$

The multiresolution representation is linked to Finite Impulse Response (FIR) filters. The scaling function ϕ and the wavelet ψ are obtained using the filter theory and consequently the coefficients are also defined by the last two equations. If at $x = t/2$, $F\{\phi(x)\}$ is considered and

$$\Phi(x) = H\left(\frac{\omega}{2}\right)\Phi\left(\frac{\omega}{2}\right) \quad (18)$$

As $\phi(0) \neq 0, H(0) = 1$, this means that $H(\omega)$ is a low-pass filter. According to this result $\phi(t)$ is computed by the low-pass fitler .The mother wavelet $\psi(t)$ is computed by defining the function $G(\omega)$ so that

$H(\omega)G^*(\omega) + H(\omega+\pi)G^*(\omega+\pi) = 0$ .Here, and $G(\omega)$ are quadrature mirror filters for the MRA solution.

$$G(\omega) = -\exp(-j\omega)H^*(\omega+\pi). \quad (19)$$

Substituting $H(0) = 1$ and $H(\pi) = 0$, it yields $G(0) = 0$ and $G(\pi) = 1$, respectively. This means that $G(\omega)$ is a high-pass filter.As a result, the MRA is a kind of Two-Channel Sub-Band Coder used in the high-pass and low-pass filters, from which the original signal can be reconstructed.

Wavelet Application on a Generated Data

In this paper, the artificial data generation in MATLAB environment is considered and deterministic signal like pure sinosoidal is generated. Here we covered the Fourier Transform based applications like Short-Time Fourier Transform and the Wavelet analysis in details.
Randomly chosen 10000 numbers (N=10000) are generated according to standard normal distribution and we used Matlab for this purpose. Randomly selected numbers are used to simulate the noisy signal. A sinusoid wavelet was generated as an artificial data that is formed of the harmonics. The main frequency is 50 Hz, second and third frequencies are assigned respectively as 100 Hz and 150 Hz. The signal, generation of these three frequency compound, is expressed as the sum of sinus and a, b, c coefficients. The generated signal in this manner is

$$y = A\sin(2\pi f_1 t) + B\sin(2\pi f_2 t) + C\sin(2\pi f_3 t), \quad (5.1)$$

where f_1 represents main frequency, f_2 and f_3 are respectively second and third harmonics.
A signal generation with noisy is carried out. The noisy signal, represented with randomly numbers, is added with a known proportion (g coefficient) to the sinusoid signal that is generated for this purpose. The sinusoid signal containing noise is represented with random numbers, with a known proportion (g coefficient).
Fourier Transform is used for the spectral analysis of the generated noisy signal. Fourier Transform is represented at PSD (Power Spectrum). Here the sampling frequency is selected to be 1000 Hz (1 kHz). Figure 1 illustrates the changes on the PSD.
Using a Short Time Fourier Transform (STFT) the same noisy signal is calculated and illustrated in a Figure 2. The STFT illustrates the signal compounds on the frequency plane. The time- frequency plane is illustrated in Figure 2; the frequency plane is a normalized plane and half of the sampling frequency 500Hz is symbolized by unit value. In Figure 2 frequency components of the signal 50, 100, 150 Hz are illustrated as spread over time plane.

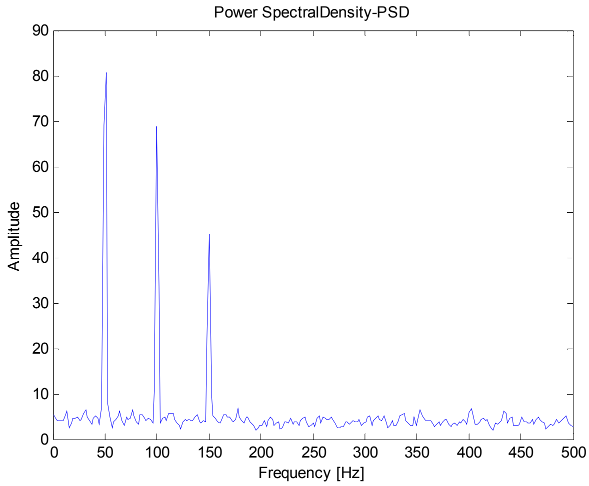

Figure 1: Spectral Analysis using PSD

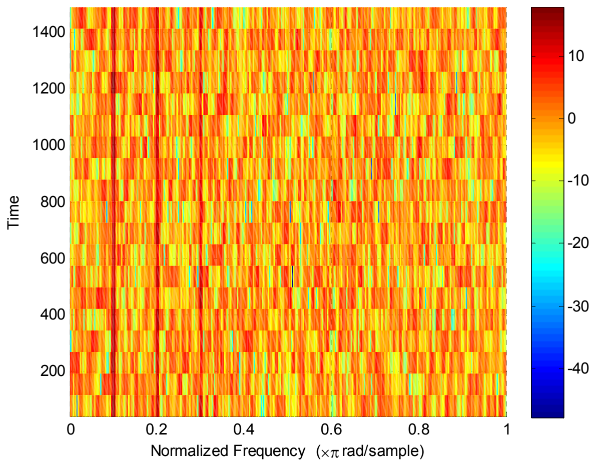

Figure 2 : Spectral Analysis using STFT

After this process, an anomaly signal is generated at random process and added to the noisy signal that is generated previously. In the application a Butterworth band pass filter. İs used and the bandwidth is taken between 200 and 250 Hz.. PSD for filter output is illustrated on figure 3. As shown on the figures the generated anomaly case contains a random signal with 200-250 Hz. The signal at the output of the filter is the anomaly case between 200-250 Hz, on the time-frequency plane it is illustrated on figure 4.

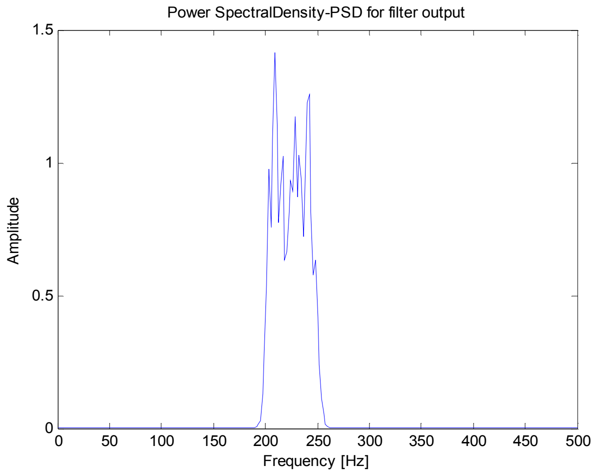

Figure 3: PSD for filter output

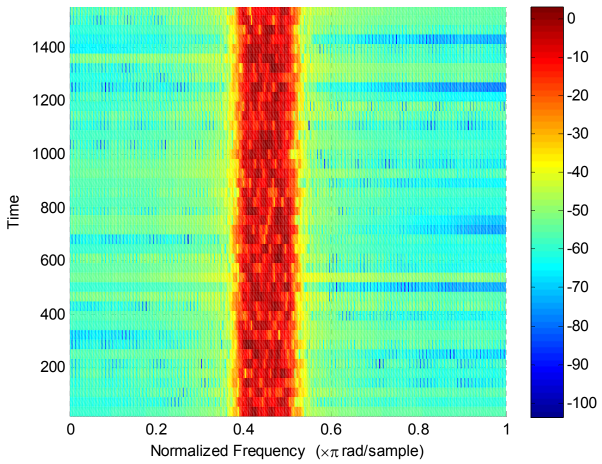

Figure 4 : Signal of anomaly (STFT of Filter Output)

After this step the anomaly case generated by the filtration is added on the previously generated y signal (sinusoidal waveform) in order to generate another new noisy signal. The difference between new noisy signal and the previous one is the anomaly case which is generated by first band pass filter is illustrated on figure 5 and anomaly between 200-250 Hz could be easily recognized.

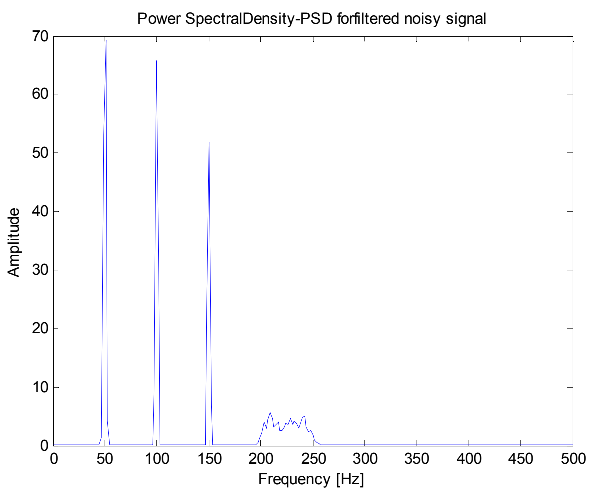

Figure 5 : PSD for Noisy Signal under the Anomally

Conclusion

In this paper, a Multi-Resolution Wavelet Analysis is used to detect the anomaly inside of the signal and then to isolate that transient case from the signal. Here we covered the Wavelet analysis in details as well as we did for the Fourier analysis. Main reason for covering both of the methods is that Fourier methods are considered as an alternative for the wavelet methods. For the detection case, we considered the Fourier Transform based applications like Short-Time Fourier Transform. It is a reperesentation of the signals in the time-frequency domain. Hence the anomally case is shown in the time-frequency plane. In terms of the isolation of the anomally case, we consiredered the multi-resolution wavelet analysis (MRWA). In this method, time-scale reperesentations of the signals are used and scales are presented by the low-pass filters (LPF) and High-Pass Filters (HPF) sequences. By sub-band analysis the anomaly case is shown in a special sub-band and it is isolated from the other sub-bands. After this isolation, the power spectral density (PSD) of the isolated sub-band is calculated and all frequency domain properties are identified as well as its statistical properties.

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