Control of a chaotic finance system with passive control

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Abstract
In this paper, complicated dynamical behavior of a finance system is investigated. The change in behavior of finance system from stable behavior to chaotic behavior is shown with varying some system parameters. In addition, chaotic finance system with passive control is considered and the stability of the controlled system is investigated. In order to control the chaos in finance system, the controller is designed based on passive control technique. Designed controller is applied to the chaotic finance system for stabilization of system. After controller is added to the system, the change in behavior of finance system from chaotic behavior to stable behavior is shown with passive control.

Keywords: Chaotic finance system, chaos control, passive control
1. INTRODUCTION

In 1963, Lorenz found the first chaotic attractor, which is named as Lorenz chaotic system, in a three dimensional autonomous system when he studied atmospheric convection (Lorenz, 1963). After Lorenz, many different chaotic systems are proposed in the past few decades such as Rössler system (OE, 1976), Chen system (G. Chen, 1999), Lü system (C.X. Liu, 2004) and finance chaotic system (Guoliang Cai, 2007). However, when chaotic behavior is sometimes undesirable, the chaotic behavior of system should be controlled. So, many methods and techniques have been developed to control the chaotic systems such as OGY method (E. Ott, 1990), sliding mode control (K. Konishi, 1998), adaptive control (Y. Zeng, 1997), and passive control (Yu, 1999; X. Chen, 2010; S. Emiroğlu, 2010).

In this paper, we study the complicated dynamic behavior and control of chaos in a nonlinear finance chaotic system which was investigated by reference (Guoliang Cai, 2007). The state equations of chaotic finance system are written below Eq 1. (Guoliang Cai, 2007)

\[
\begin{align*}
\dot{x} &= z + (y - a)x \\
\dot{y} &= 1 - by - x^2 \\
\dot{z} &= -x - cz
\end{align*}
\]

where variable \( x \) represents the interest rate in the model; variable \( y \) represents the investment demand and variable \( z \) is the price exponent. The parameter \( a \) is the saving, \( b \) is the per-investment cost, \( c \) is the elasticity of demands of commercials. And they are positive constants.

Mathematical model of a finance system is constructed by using Matlab-Simulink program as shown in Figure 1.

![Figure 13 Matlab-Simulink model of finance system](image)

By using Matlab-Simulink model of finance system, chaotic time series and phase portraits of the system is shown in Figure 2.
2. THE THEORY OF PASSIVE CONTROL

Consider a nonlinear system (2) modelled by ordinary differential equation with input vector $u(t)$ and output vector $y(t)$ (Yu, 1999),

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u, \\
y &= h(x),
\end{align*}
\]

where the state variable $x \in \mathbb{R}^n$, the input $u \in \mathbb{R}^m$ and the output $y \in \mathbb{R}^m$. $f(x)$ and $g(x)$ are smooth vector fields. $h(x)$ is a smooth mapping. We suppose that the vector field $f$ has at least one equilibrium point and without loss of the generality, we assume the equilibrium point $x=0$.

Definition 1. System (2) is a minimum phase system if $Lgh(0)$ is nonsingular and $x=0$ is one of the asymptotically stabilized equilibrium points of $f(x)$.

Definition 2. System (2) is passive if the following two conditions are satisfied:

1. $f(x)$ and $g(x)$ exist and are smooth vector fields, $h(x)$ is also a smooth mapping.
2. For any $t \geq 0$, there is a real value $\beta$ that satisfies the inequality

Figure 14 Phase portraits of the system
\[
\int_0^t u^T(\tau) y(\tau) d\tau \geq \beta, \quad (3)
\]

or there are real values \( \beta \) and \( \rho \geq 0 \) that satisfy the inequality

\[
\int_0^t u^T(\tau) y(\tau) d\tau + \beta \geq \int_0^t \rho y^T(\tau) y(\tau) d\tau, \quad (4)
\]

When we let \( z = \Phi(x) \) system (2) can be changed into the following generalized form

\[
\begin{align*}
\dot{z} &= f_0(z) + p(z, y) y, \\
\dot{y} &= b(z, y) + a(z, y) u,
\end{align*} \quad (5)
\]

where \( a(z, y) \) is nonsingular for any \((z, y)\).

If system (2) has relative degree \([1, 1, ...]\) at \( x = 0 \) and system (1) is a minimum phase system, then system (5) will be equivalent to a passive system and will be asymptotically stable at equilibrium points through the local feedback control as follows:

\[
u = a(z, y)^{-1} [-b^T(z, y) - \frac{\partial W(z)}{\partial z} p(z, y) - \alpha y + v]
\]

where \( W(z) \) is the Lyapunov function of \( f_0(z) \), \( \alpha \) is a positive real value, and \( v \) is an external signal which is connected to the reference input.

### 3. CHAOS CONTROL OF CHAOTIC FINANCE SYSTEM

In this section, the control of chaotic system (7) is achieved using passive control theory. The controlled model given by

\[
\begin{align*}
\dot{x} &= z + (y - a)x \\
\dot{y} &= 1 - by - x^2 + u \\
\dot{z} &= -x - cz
\end{align*} \quad (7)
\]

The controller is designed based on passive control theory (Yu, 1999). The controller is shown in Eq. 8 and also controlled system is written in Eq. 7.

\[
u = -1 + y(b - \alpha) + v \quad (8)
\]

Time series of system and controlled system are shown in Fig. 3. After the controller is activated at \( t=300s \), the system converges to zero equilibrium point as shown in Fig. 3.
4. CONCLUSION

We investigate chaos control of a 3D chaotic finance system via passive control method in this paper. Based on the passive system theory, passive controller is proposed to realize the global asymptotical stability of the 3D chaotic finance system. Finally, numerical simulations are provided to verify the theoretical analysis and also show that the proposed method works effectively.

REFERENCES


Synchronization of a chaotic finance system via active control

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Abstract
This paper discusses chaos synchronization of the three dimensional finance system based on active control technique. Using active control theory, chaos synchronization of three dimensional chaotic finance system is realized with three input. The designed controllers ensure the stability of error dynamical system between two identical chaotic finance systems. Also, the controllers provide that the error dynamical system converges to zero equilibrium. Numerical simulations show that the proposed method is effective for chaotic finance system.

Keywords: Chaotic finance system, chaos synchronization, active control

1. INTRODUCTION
Since the control of chaotic systems is firstly proposed by Ott, Grebogi and Yorke, chaos control has become one of the much interesting research subject. Also, chaos synchronization has received a huge increasing interest and has been studied in the past two decades, after Pecora and Carroll introduced the synchronization method. Recently, many control methods are proposed to the control and the synchronization of the chaotic systems. The control strategies applied to control and synchronization of chaos such as OGY method (E. Ott, C. G. 1990), linear feedback control (A.E. Matouk, 2008), passive control (S. Emiroğlu and Y. Uyaroğlu, 2010; X. Chen, C. L. 2010), active control (S. Emiroğlu, Y. Uyaroğlu, 2011) etc.