A Critique On The Consistency Ratios Of Some Selected Articles Regarding Fuzzy AHP And Sustainability

Bülent Başaran

Affiliation: Bilecik University, the Faculty of Economics and Administrative Sciences, Bilecik, Turkey

Abstract

Consistency ratio (CR) is a very important indicator for achieving the reliability of an individual’s pairwise comparisons in Analytic Hierarchy Process (AHP). Although the applications of fuzzy AHP need this kind of CR results as well, only a few of studies include these results. The most accepted method to calculate CR for fuzzy pairwise comparison matrices (PCMs) is to transform fuzzy numbers to crisp versions and to proceed as in the ordinary CR calculations of AHP. Triangular fuzzy numbers (TFNs) are usually used to present linguistic terms of an individual’s pairwise comparisons. In this research, CRs of 242 PCMs presented with TFNs, found in 39 articles, have been calculated based on four widely used defuzzification methods. The aim of this research is to find out if the PCMs of some available articles regarding sustainability issues in literature are reliable. After CR calculations of those PCMs, it has been found that some of them are reliable while many others are not. After reviewing these findings, researchers in fuzzy AHP field are expected to give much attention to those CR issues and try to obtain PCMs that are more reliable.
Keywords: fuzzy AHP, consistency ratio, sustainability, defuzzification, fuzzy numbers

1. INTRODUCTION

Analytic Hierarchy Process (AHP), developed by Saaty (1980), is a very powerful multicriteria prioritization technique. However, its applicability is not only limited to the number of criteria but also to the consistency ratio (CR) of pairwise comparison matrices (PCMs). If the CR value of a PCM passes 0.1, it means that the matrix is not consistent and comparisons should be made again in order to have a reasonable CR value.

Fuzzy AHP is developed because of the fuzzy nature of those pairwise comparisons (Chang 1996). To provide more fuzziness (according to Saaty and Tran’s ‘2007’ criticism), usually triangular fuzzy numbers (TFNs) are used in pairwise comparisons. Although some kind of CR value is needed to evaluate those PCMs in fuzzy AHP as well, many of the research articles skip this evaluation. The aim of this study is to present how much the results of those researches are reliable when taking CR issues into consideration. A total of 242 PCMs, found in 39 articles, have been examined in terms of their crisp version CR values.

The paper is organized as follows: first, the applications of fuzzy AHP in literature are described; second, some defuzzification methods for obtaining the crisp versions of those TFNs are explained; third, how TFNs differ in their crisp versions under different diffuzzification methods are shown; fourth, CR values of PCMs in some selected articles are evaluated. Finally, remarkable conclusions and some future directions are given.

2. FUZZY AHP AND ITS APPLICATIONS IN LITERATURE

Fuzzy AHP is one of the most widely used methods in multicriteria decision making although Saaty and Tran (2007) criticizes it seriously about its fuzzifying judgments. Some applications of fuzzy AHP exist in the field of sustainability and sustainable developments. Although there are some fuzzy approaches to obtain a priority weight vector (Chang 1996, Liou and Wang 1992), Chang’s extent synthesis method is used in many fuzzy AHP studies.

Among many research areas regarding sustainability and fuzzy AHP, some of them can be given as follows:

- Supplier or firm selection: Kahraman et al. (2004), Chan et al. (2008), Efendigil et al. (2008), Lee et al. (2009), Şen et al. (2010), Chen et al. (2011), Pei et al. (2011), Aydin et al. (2012), Öztürk and Başkaya (2012).
- Production process selection: Talinli et al. (2010).
- Personnel selection: Celik et al. (2009), Pei et al. (2011), Bulut et al. (2012).
- Quality issues: Kwong and Bai (2003), Büyüközkan et al. (2011), Aydin et al. (2012).
Environmental issues: Lee et al. (2009), Tseng et al. (2009), Zheng et al. (2010), Karimi et al. (2011), Wang et al. (2011).

3. DEFUZZIFICATION OF TRIANGULAR FUZZY NUMBERS

A TFN can be defined by the membership function (1) (Bulut et al. 2012).

\[
\mu_A(x) = \begin{cases} 
0, & x < l, \\
(x-l)/(m-l), & l \leq x < m, \\
1, & x = m, \\
(u-x)/(u-m), & m < x \leq u, \\
0, & u < x, 
\end{cases}
\] (1)

where \( l \) and \( u \) correspond to the lower and upper bounds of the fuzzy number \( A \), respectively, and \( m \) is the midpoint. The TFN is indicated as \( A = (l, m, u) \).

The methods for defuzzification of TFNs used in this study are as follows:

- **Weighted Mean Method:** According to Kwong and Bai (2003), a TFN can be defuzzified to a crisp number by equation (2).

\[
\tilde{A}_{\text{crisp}} = \frac{(l+4m+u)}{6}
\] (2)

This method has been used by some researchers for transforming TFNs to crisp versions and calculated CRs accordingly. Although equation (2) is very easy to calculate, it may cause some bias problems because of weighting. It is more appropriate for the TFNs shaping as an equilateral triangle. However, especially the reciprocals of TFNs used in fuzzy PCMs do not shape as an equilateral triangle most of the time.

- **Centroid Method:** It is also called “center of gravity” method and is the most widely used one among other defuzzification methods. Centroid defuzzification returns the center of area under the curve as in equation (3).

\[
\tilde{A}_{\text{crisp}} = \frac{\int \mu_A(x) x dx}{\int \mu_A(x) dx}
\] (3)

For TFNs as in function (1), the result of centroid method will be equal to \((l + m + u)/3\).

- **Bisector Method:** The bisector is the vertical line that will divide the region into two sub-regions of equal area. It will be equal to \( m \) for equilateral TFNs.

- **Middle, Smallest, and Largest of Maximum (MOM, SOM, and LOM) Methods:** Since any TFN used in a PCM has a unique maximum, the result of these three methods will be the same. Therefore, only the MOM results have been shown in this study.
4. VARIATIONS IN TRIANGULAR FUZZY NUMBERS

The TFNs used for importance degrees are equilateral linguistic variables except for the highest level. In some researches such as Zheng et al. (2010), Zheng (2011), Wang et al. (2011), Tseng et al. (2009), Toksarı and Toksarı (2011), Bozbura and Beskese (2007), Ahari et al. (2011), Büyükozkan et al. (2011), the highest level is also equilateral. However, in many other researches, the highest level is not equilateral. On the other hand, reciprocal values of any TFNs are not equilateral in almost every time. Because of these reasons, weighted mean method is not the best approach in many situations and researchers need to look at how some other defuzzification methods will result. Table 1 shows how crisp versions of some reciprocal TFNs can differ in terms of different defuzzification techniques.

Table 1. Defuzzification of TFNs

<table>
<thead>
<tr>
<th>Importance Degrees*</th>
<th>TFNs</th>
<th>Weighted Mean</th>
<th>Centroid</th>
<th>Bisector</th>
<th>MOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equally important</td>
<td>(1/2, 1, 3/2)</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>Weakly more important</td>
<td>(1, 3/2, 2)</td>
<td>1.5000</td>
<td>1.5000</td>
<td>1.5000</td>
<td>1.5000</td>
</tr>
<tr>
<td>Strongly more important</td>
<td>(3/2, 2, 5/2)</td>
<td>2.0000</td>
<td>2.0000</td>
<td>2.0000</td>
<td>2.0000</td>
</tr>
<tr>
<td>Very strongly more important</td>
<td>(2, 5/2, 3)</td>
<td>2.5000</td>
<td>2.5000</td>
<td>2.5000</td>
<td>2.5000</td>
</tr>
<tr>
<td>Absolutely more important</td>
<td>(5/2, 3, 7/2)</td>
<td>3.0000</td>
<td>3.0000</td>
<td>3.0000</td>
<td>3.0000</td>
</tr>
</tbody>
</table>

Reciprocals

<table>
<thead>
<tr>
<th>Importance Degrees*</th>
<th>TFNs</th>
<th>Weighted Mean</th>
<th>Centroid</th>
<th>Bisector</th>
<th>MOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reciprocal of equally important</td>
<td>(2/3, 1, 2)</td>
<td>1.1111</td>
<td>1.2222</td>
<td>1.1835</td>
<td>1.0000</td>
</tr>
<tr>
<td>Reciprocal of weakly more important</td>
<td>(1/2, 2/3, 1)</td>
<td>0.6944</td>
<td>0.7222</td>
<td>0.7113</td>
<td>0.6667</td>
</tr>
<tr>
<td>Reciprocal of strongly more important</td>
<td>(2/5, 1/2, 2/3)</td>
<td>0.5111</td>
<td>0.5222</td>
<td>0.5176</td>
<td>0.5000</td>
</tr>
<tr>
<td>Reciprocal of very strongly more important</td>
<td>(1/3, 2/5, 1/2)</td>
<td>0.4056</td>
<td>0.4111</td>
<td>0.4087</td>
<td>0.4000</td>
</tr>
<tr>
<td>Reciprocal of absolutely more important</td>
<td>(2/7, 1/3, 2/5)</td>
<td>0.3365</td>
<td>0.3397</td>
<td>0.3383</td>
<td>0.3333</td>
</tr>
</tbody>
</table>


5. EVALUATIONS OF SOME SELECTED ARTICLES

A total of 39 articles regarding fuzzy AHP and sustainability have been taken from the literature. There are 242 PCMs developed with TFNs in those articles. A MATLAB m_file has been coded to compute all of those PCMs’ CRs according to above mentioned four defuzzification methods. Table 2 shows just two of those articles’ calculation results. It is not
possible to show all CR values for all PCMs in this space limited study. On the other hand, the articles and their number of PCMs that have CR values above 0.1 are shown in Table 3.

Table 2. Some examples of exact CR calculation results

<table>
<thead>
<tr>
<th>Articles</th>
<th>PCMs Presented with TFNs (Matrix No.)</th>
<th>CR Values based on The Following Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Weighted Mean</td>
</tr>
<tr>
<td>Chen et al. (2011)</td>
<td>1</td>
<td>0.0432</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0965</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.1566</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.3750</td>
</tr>
<tr>
<td>Lee et al. (2009)</td>
<td>1</td>
<td>0.0761</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.1573</td>
</tr>
</tbody>
</table>

Table 3. CR results of matrices in literature

<table>
<thead>
<tr>
<th>Articles</th>
<th>Number of PCMs Presented with TFNs</th>
<th>Number of PCMs whose Crisp Versions Have a CR above 0.1 based on The Following Defuzzification Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Weighted Mean</td>
</tr>
<tr>
<td>Ahari et al. (2011)</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Aydin et al. (2012)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bulut et al. (2012)*</td>
<td>25</td>
<td>14</td>
</tr>
<tr>
<td>Büyükozkkan et al. (2011)*</td>
<td>26</td>
<td>-</td>
</tr>
<tr>
<td>Celik et al. (2009)</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Chaghooshi et al. (2011)*</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Chan et al. (2008)</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>Chen et al. (2011)</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Deng and Molla (2008)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Efendigil et al. (2008)</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
6. CONCLUSIONS

Many researchers continue to use fuzzy AHP without any explanation and calculation about CRs of PCMs used in their studies. Although the methods and applications submitted in their researches give valuable contributions to literature, violation of the CR limit may decrease the reliability of their currently published articles.

In this study, after a broad review of literature, 242 PCMs in 39 articles have been examined. The TFNs have been transformed into their crisp versions via four different defuzzification methods and the CRs of those PCMs have been calculated as in the ordinary AHP. The results have shown that many of those PCMs violated the 0.1 upper-limit under some defuzzification methods while they did not violate this limit under some others. Because of this reason, researchers in fuzzy AHP field need to pay much attention to the CR issues for their PCMs. More researches are needed to formulate a more suitable CR for fuzzy AHP applications.

REFERENCES


---

**Using Artificial Neural Networks To Forecast Gdp For Turkey**

Karaatli Meltem, Göçmen Yağcilar Gamze, Karacadal Hüseyin, Sezer Fırat Suleyman

*Suleyman Demirel University, Isparta, Turkey*

E-mails: meltemkaraatli@sdu.edu.tr, gamzeyagcilar@sdu.edu.tr,

huseyin_karacadal@hotmail.com, cihangir_07_@hotmail.com

**Abstract**

Artificial Neural Networks (ANN) is a system resembling biological neural systems and uses working principles of human brain as a base. ANN can be applied in various fields for the purposes of forecasting, classification, optimization, data binding and so on. ANN has been frequently used in financial applications in recent years. In this study, ANN is used in forecasting Gross Domestic Product of Turkey. Gross Domestic Product (GDP) refers to the market value of all final goods and services produced within a country in a given period. GDP can be thought as the size of an economy and it is the foremost important measure of macroeconomic performance of a country, a country’s health and standard of living. Therefore, expectations about future GDP can be the primary determinant of investments, employment, wages, profits and even stock market activities. With respect to its economic